

Name \_\_\_\_\_

AR 5

## Common Factors and Simplest Form

**Essential Question** How can you find the greatest common factor of two numbers?

**NO.3.5.5** Use *factors* of numbers: to introduce exponents, to find common *factors*, and to simplify fractions

### UNLOCK the Problem

REAL WORLD

Jim is cutting two strips of wood to make square picture frames that are all the same size. The wood strips are 8 feet and 24 feet. For Jim to have no wood left over, into what whole-number lengths can he cut the wood strips so that all lengths are the same? Find all of the different lengths into which he can cut the strips of wood.

8 feet

24 feet

#### Remember

A number that is multiplied by another number to find a product is a factor.

Factors of 6: 1, 2, 3, 6

Factors of 9: 1, 3, 9

Every number has 1 as a factor.

**Find the common factors of 8 and 24.**

A **common factor** is a number that is a factor of two or more numbers.

**STEP 1** List all the factors of each number.

**Think:** In what ways can each wood strip be cut into equal pieces?

Factors of 8: \_\_\_\_\_

Factors of 24: \_\_\_\_\_

**STEP 2** Identify the common factors.

**Think:** Into what equal lengths can both wood strips be cut?

Common factors of 8 and 24: \_\_\_\_\_

So, Jim could cut both strips of wood into lengths of \_\_\_\_\_ feet.

### Try This!

**What if** Jim wants to cut the two strips of wood into equal lengths that are as long as possible? Into what length should he cut the wood?

The **greatest common factor**, or **GCF**, is the greatest factor that two or more numbers have in common.

The common factors of 8 and 24 are \_\_\_\_\_.

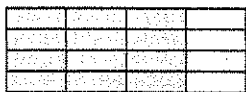
The greatest common factor of 8 and 24 is \_\_\_\_\_.

So, Jim should cut the wood into \_\_\_\_\_-foot strips.

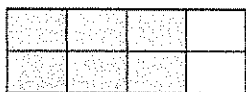
**Common Factors and Simplest Form** Common factors and the GCF can help you work with fractions.

A fraction is in **simplest form** when the numerator and the denominator both have 1 as their only common factor.

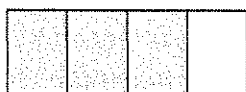
**One Way** Write  $\frac{12}{16}$  in simplest form.



$$\frac{12}{16}$$



$$\frac{6}{8}$$



Simplest form: \_\_\_\_\_

**Other Ways**

You can find the simplest form of a fraction by dividing both the numerator and the denominator at least once by a common factor or by dividing just once by the greatest common factor.

**A** Use a common factor.

**STEP 1**

Divide the numerator and denominator by a common factor.

Think: 12 and 16 are both even numbers, so 2 is a common factor.

$$\frac{12 \div 2}{16 \div 2} = \frac{6}{8}$$

**STEP 2**

Continue dividing by a common factor until the numerator and denominator have no common factor except 1.

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

**B** Use the greatest common factor.

**STEP 1**

List the factors of each number. Then find the GCF.

Factors of 12: \_\_\_\_\_

Factors of 16: \_\_\_\_\_

The GCF is \_\_\_\_\_.

**STEP 2**

Divide the numerator and denominator by their GCF.

$$\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

**Math Talk**

When you divide both the numerator and denominator of a fraction by the same number, the value of the fraction does not change. Explain why.

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## Share and Show



List the common factors for each pair of numbers.

Circle the greatest common factor.

1. 12, 20

Think: Factors of  
12: 1, 2, 3, 4, 6, 12

Factors of 20: \_\_\_\_?

2. 15, 18

3. 16, 24

4. 9, 27

Write each fraction in simplest form.

5.  $\frac{10}{30}$  \_\_\_\_\_

6.  $\frac{4}{22}$  \_\_\_\_\_

**Math Talk**

Explain how you  
would write the fraction  $\frac{14}{35}$  in  
simplest form.

## On Your Own

List the common factors for each pair of numbers.

Circle the greatest common factor.

7. 25, 50

8. 28, 35

9. 30, 40

10. 48, 56

Name the GCF of the numerator and the denominator.

Then write each fraction in simplest form.

11.  $\frac{6}{16}$

12.  $\frac{24}{32}$

13.  $\frac{18}{30}$

14.  $\frac{20}{100}$



Find the missing value.

15.  $\frac{3}{4} = \frac{\square}{16}$

$\square =$  \_\_\_\_\_

16.  $\frac{1}{2} = \frac{16}{\square}$

$\square =$  \_\_\_\_\_



17.  $\frac{\square}{3} = \frac{18}{27}$

$\square =$  \_\_\_\_\_

# Problem Solving

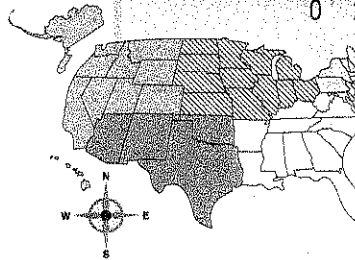
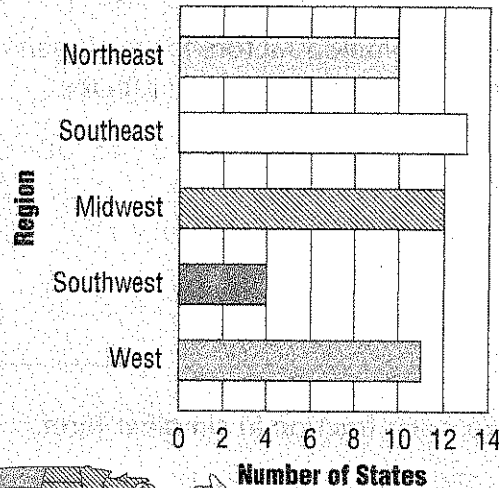
REAL WORLD

Use the graph to solve 18–20 and 22.

18. What fraction of the 50 states does the Southeast region represent? Write your answer in simplest form.  
  
\_\_\_\_\_
19. What fraction of the 50 states does the Northeast region represent? Write your answer in simplest form.  
  
\_\_\_\_\_
20. What fraction of the 50 states do the West and Southwest regions together represent? Write your answer in simplest form.  
  
\_\_\_\_\_
21.  Florida borders both the Atlantic Ocean and the Gulf of Mexico. Thirteen states border only the Atlantic Ocean. Four other states border only the Gulf of Mexico. Use simplest form to write the fraction of the 50 states that border one or both of these bodies of water.  
  
\_\_\_\_\_
22.  **Test Prep** What fraction of the 50 states are not Midwestern states?

- (A)  $\frac{6}{25}$   
 (B)  $\frac{12}{50}$   
 (C)  $\frac{12}{25}$   
 (D)  $\frac{19}{25}$

Regions of the United States



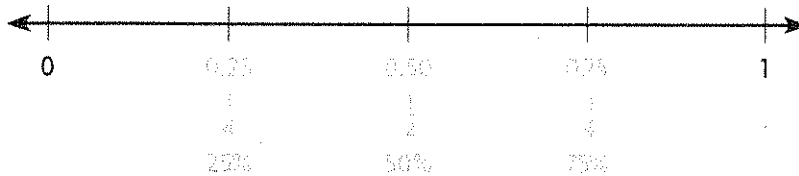
SHOW YOUR WORK

Name \_\_\_\_\_

## Fractions, Decimals, and Percents

**Essential Question** How do fractions, decimals, and percents relate to each other?

**CONNECT** You can use what you know about benchmarks to relate fractions, decimals, and percents. A **benchmark** is a familiar number used as a point of reference.



**NO.1.5.1** Use models and visual representations to develop the concepts of fractions, ratios, and percents

**NO.1.5.3** Identify decimal and percent equivalents for benchmark fractions

### UNLOCK the Problem

### REAL WORLD

In the United States,  $\frac{1}{20}$  of the forests are reserved forests, not harvested for industry. What percent of the forests in the United States are reserved forests?

**Write a fraction as a percent.**

**STEP 1** Since *percent* means "per hundred," write an equivalent fraction with a denominator of 100.

$$\frac{1}{20} = \frac{1 \times \boxed{\phantom{00}}}{20 \times \boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

**STEP 2** Write the numerator as a percent.

**Think:** There are 5 parts per hundred.

\_\_\_\_\_ %

So, \_\_\_\_\_ of the forests in the United States are reserved forests.



- The fraction  $\frac{20}{100}$  can be read as twenty hundredths and expressed as a decimal, 0.20. What decimal can you write for the part of the forests in the United States that is reserved forests? Explain your reasoning.

**Math Talk** Explain how you would write the fraction  $\frac{8}{10}$  as a percent and as a decimal using word form and place value.

**Relate Percents and Decimals to Fractions** You can write a percent as a fraction or a decimal.

## Example 1

**A** Write 60% as a fraction in simplest form.

- Write the percent as a fraction, using a denominator of 100.

$$60\% = \frac{\quad}{100}$$

- Write the fraction in simplest form.

$$\frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

Think: The GCF for 60 and 100 is 20.

**B** Write 33% as a decimal.

- Write the percent as a fraction, using a denominator of 100.

$$33\% = \frac{\quad}{100}$$

- Write the fraction as a decimal.

$$\frac{33}{100} = 0.33$$

Think: Read the fraction and use place value to write the decimal.

2. Suppose there are 100 marbles in a bag and you pull 15% of them from the bag without looking. Explain how you could write a fraction and a decimal for the number of marbles pulled.
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## Example 2

**A** Write 0.125 as a fraction in simplest form.

- Use place value to express the decimal as a fraction.

$$0.125 = \frac{125}{1,000}$$

Think: 0.125 is one hundred twenty-five thousandths.

- Write the fraction in simplest form.

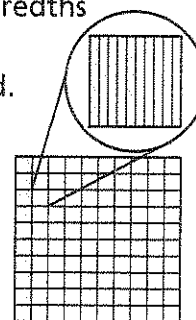
$$\frac{125}{1,000} = \frac{125 \div 25}{1,000 \div 25} = \frac{5}{40} = \frac{1}{8}$$

**B** Write 0.125 as a percent.

- The model shows 12 hundredths shaded, and 5 tenths of a hundredths square shaded.

Think: 12.5 of 100 squares are shaded.

$$\underline{\quad\quad\quad}\%$$



3. Explain how the model in Example 2B helps you understand how to write a decimal in thousandths as a percent.
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Name \_\_\_\_\_

# Share and Show



1. Write  $\frac{2}{5}$  as a percent.

$$\frac{2}{5} = \frac{2 \times \quad}{5 \times \quad} = \frac{\quad}{100}$$

So, there are \_\_\_\_\_ parts per hundred, or \_\_\_\_\_%.

Complete the table. Write each fraction in simplest form.

	Fraction	Decimal	Percent
2.	$\frac{6}{10}$	0.60	
3.		0.24	24%
4.	$\frac{4}{5}$	0.80	

	Fraction	Decimal	Percent
5.		0.18	18%
6.	$\frac{3}{25}$		
7.		0.75	

## Math Talk

Explain why

$\frac{5}{10}$  is not equal to 5%.

# On Your Own

Complete the table. Write each fraction in simplest form.

	Fraction	Decimal	Percent
8.	$\frac{1}{2}$	0.50	
9.	$\frac{3}{20}$		15%
10.		0.625	62.5%
11.		0.85	85%
12.	$\frac{5}{5}$	1	

	Fraction	Decimal	Percent
13.		0.875	87.5%
14.	$\frac{1}{5}$	0.20	
15.	$\frac{7}{20}$		
16.			52%
17.		0.95	

# Problem Solving

REAL WORLD

Use the table for 18–20.

18. What fraction can you write for the percent of paper recovered by recycling in the United States each year? Write the fraction in simplest form.

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19. About what percent of the hardwood forests in the United States are not privately owned? Write the percent as a fraction in simplest form.

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20. **NOTE** The amount of land in the United States that is covered by forest is about 0.33. What decimal can you write to estimate the amount of land that is not covered by forest? Explain how you solved this problem.

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21. Jill has a number cube labeled 2, 4, 5, 6, 7, and 9. Write a fraction in simplest form, a decimal, and a percent for the number of sides labeled with an even number.

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22. Robert has two spinners. One spinner has  $\frac{1}{4}$  of the sections colored red. The second spinner has 40% of the sections colored red. What percent more does the second spinner have colored red?

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23. **★ Test Prep** Kay correctly answered 85% of the questions on a math test. What is that percent written as a decimal?

- (A) 0.85                      (C) 85  
(B) 8.5                        (D) 850

## Forestry Facts

- The forest covers  $\frac{1}{3}$  of all land in the United States.
- More than 72% of hardwood forests in the United States are privately owned.
- Each year 1.4 billion tree seedlings are planted, or about 4 million per day.
- About 45% of paper consumed in the United States is recovered by recycling.

## SHOW YOUR WORK



Name \_\_\_\_\_

# Ratios and Rates

**Essential Question** How do you write ratios and rates?

**NO.1.5.1** Use models and visual representations to develop the concepts of fractions, ratios, and percents

## UNLOCK the Problem REAL WORLD

A bird rescue group is caring for 3 eagles, 2 hawks, and 5 owls in their rescue center.

You can compare the numbers of different types of birds using ratios. A **ratio** is a comparison of two numbers,  $a$  and  $b$ , that can be written as a fraction  $\frac{a}{b}$ . There are three ways to write the ratio of owls to eagles in the rescue center.

Using words      As a fraction      With a colon

5 to 3

$\frac{5}{3}$

5:3

Ratios can be written to compare a part to a part, a part to a whole, or a whole to a part.

**Write each ratio using words, as a fraction, and with a colon.**

**A** Owls to hawks

_____ to _____		_____ : _____	Part to part
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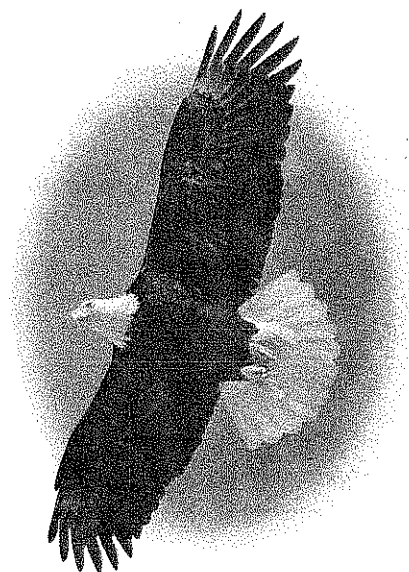
**B** Eagles to total birds in the rescue center

_____ to _____		_____ : _____	Part to whole
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**C** Total birds in the rescue center to hawks

_____ to _____		_____ : _____	Whole to part
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- The ratio of owls to total number of birds is 5:10.  
Explain what this ratio means.





**Example** A restaurant sells veggie burgers at the rate of \$4

for 1 burger. What rate gives the cost of 5 veggie burgers? Write the rate for 5 burgers using words, as a fraction, and with a colon.

A **rate** is a ratio that compares two quantities that have different units of measure.

A **unit rate** is a rate that makes a comparison to 1 unit. The unit rate for cost per veggie burger is \$4 to 1 burger or  $\frac{\$4}{1 \text{ burger}}$ .

Complete the table to find the rate that gives the cost of 5 veggie burgers.

Think: 1 veggie burger costs \$4, so 2 veggie burgers cost  $\$4 + \underline{\hspace{2cm}}$ , or  $2 \times \underline{\hspace{2cm}}$ .

	Unit Rate	$2 \cdot \$4$ ↓	$3 \cdot \$4$ ↓	$\text{ } \cdot \$4$ ↓	$\text{ } \cdot \$4$ ↓
<b>Cost</b>	\$4	\$8			
<b>Veggie Burgers</b>	1	2	3	4	
		↑ $2 \cdot 1$	↑ $\text{ } \cdot 1$	↑ $4 \cdot 1$	↑ $\text{ } \cdot 1$

The table shows that 5 veggie burgers cost  $\underline{\hspace{2cm}}$ .

So, the rate that gives the cost for 5 veggie burgers is

\$ $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$  burgers,  $\frac{\$ \text{ } \text{ }}{\text{ } \text{ burgers}}$ , or \$ $\underline{\hspace{2cm}}$  :  $\underline{\hspace{2cm}}$  burgers.

**MATHEMATICAL PRACTICES**

**Math Talk**

Describe two other ways to say "\$4 per burger".

**Try This!** Write the rate in three different ways.

**A** The rate that gives the cost of 3 veggie burgers

$\underline{\hspace{5cm}}$

**B** The rate that gives the cost of 4 veggie burgers

$\underline{\hspace{5cm}}$

2. Explain why the ratio  $\frac{\$4}{1 \text{ burger}}$  is a unit rate.

$\underline{\hspace{10cm}}$

3. Explain the pattern you see in the table in the Example.

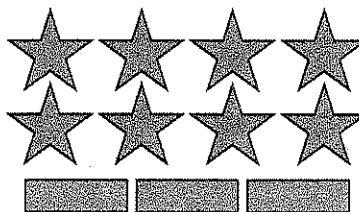
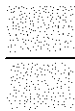
$\underline{\hspace{10cm}}$

$\underline{\hspace{10cm}}$

Name \_\_\_\_\_

# Share and Show

1. Write the ratio of the number of bars to stars.



Write the ratio in two different ways.

2. 8 to 16

3.  $\frac{4}{24}$

4. 1:3

5. 7 to 9

6. Marilyn saves \$15 per week. Complete the table to find the rate that gives the amount saved in 4 weeks. Write the rate in three different ways.

Savings		\$30	\$45		\$75
Weeks	1	2	3	4	5

**Math Talk** MATHEMATICAL PRACTICES  
Explain whether the ratios 5:2 and 2:5 are the same or different.

## On Your Own

Write the ratio in two different ways.

7.  $\frac{16}{40}$

8. 8:12

9. 4 to 11

10. 2:13

11. There are 24 baseball cards in 4 packs. Complete the table to find the rate that gives the number of cards in 2 packs. Write this rate in three different ways.

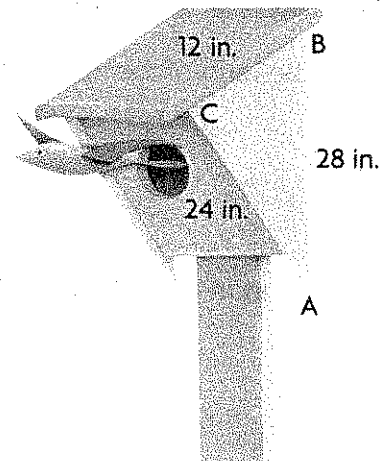
Cards			18	24
Packs	1	2	3	4

12. Explain how the statement "There are 6 apples per bag" represents a rate.

# Problem Solving

REAL WORLD

Use the diagram of a birdhouse for 13–15.



13. Write the ratio of  $AB$  to  $BC$  in three different ways.  
\_\_\_\_\_
14. Write the ratio of the shortest side length of triangle  $ABC$  to the perimeter of the triangle in three different ways.  
\_\_\_\_\_
15. Write the ratio of the perimeter of triangle  $ABC$  to the longest side length of the triangle in three different ways.  
\_\_\_\_\_

16. Leandra places 6 photos on each page in a photo album. Find the rate that gives the number of photos on 2 pages. Write the rate in three different ways.  
\_\_\_\_\_

17. **NOT** What's the Question? The ratio of total students in Ms. Murray's class to students in the class who have an older brother is 3 to 1. The answer is 1:2. What is the question?  
\_\_\_\_\_  
\_\_\_\_\_

18. **Write Math** What do all unit rates have in common?  
\_\_\_\_\_  
\_\_\_\_\_

19. **★ Test Prep** Two smoothies at Smoothie Haven cost \$6. What is the unit rate for the cost per smoothie at Smoothie Haven?

- (A)  $\frac{\$1}{3 \text{ smoothies}}$   
 (B)  $\frac{\$6}{2 \text{ smoothies}}$   
 (C)  $\frac{\$3}{1 \text{ smoothie}}$   
 (D)  $\frac{\$6}{1 \text{ smoothie}}$

SHOW YOUR WORK